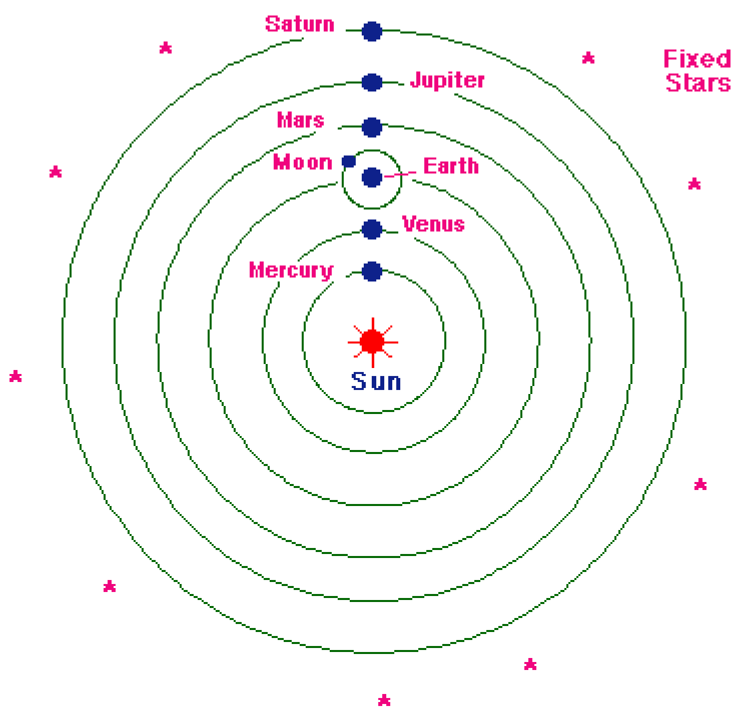
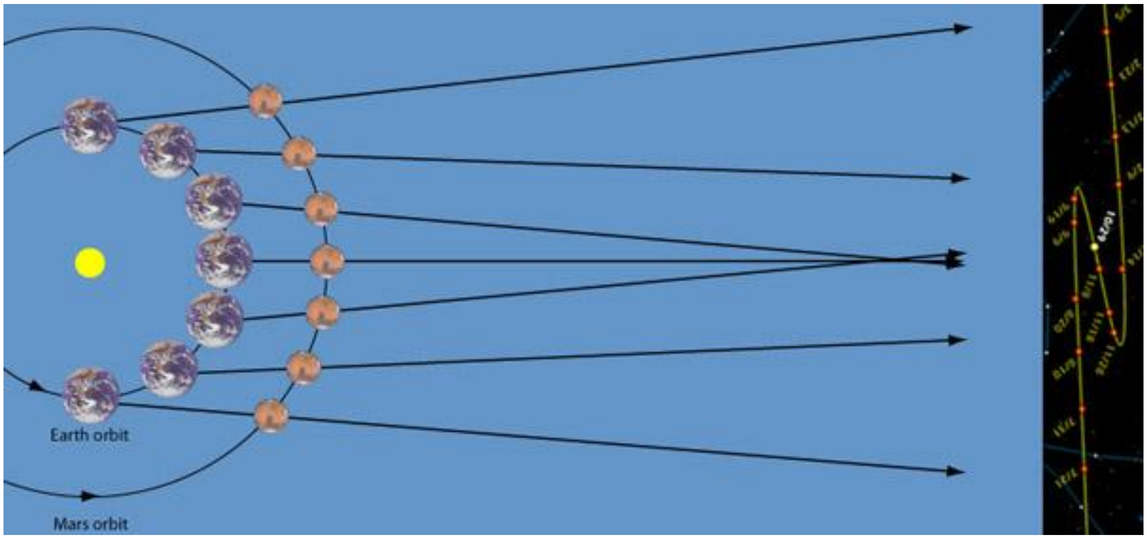
VII. Astronomy Revolution:

A. A breakthrough in our understanding of the Solar System occurred when the Polish astronomer Nicolaus Copernicus advance his Sun-centered ("heliocentric") theory of planetary motions.

1. His idea had been considered earlier, but it did not have a lasting effect, and surviving records are fragmentary.
    a. Aristarchus, in particular, had suggested a heliocentric model around 250 B.C.
    b. We know about this due to Archimedes' work, *The Sand Reckoner*. Any original manuscript burned during the great fire in the library of Alexandria.
2. With the Sun at the center, and planets orbiting it, retrograde motion could be explained



a. Mars, for example, normally appear to move from west to east among the stars.
b. However, Mars takes longer to orbit the Sun than Earth does.
c. Earth therefore sometimes overtakes Mars in it's orbit, and the perspective changes.
d. During such times, Mars appears to move backwards (east to west) among the stars.

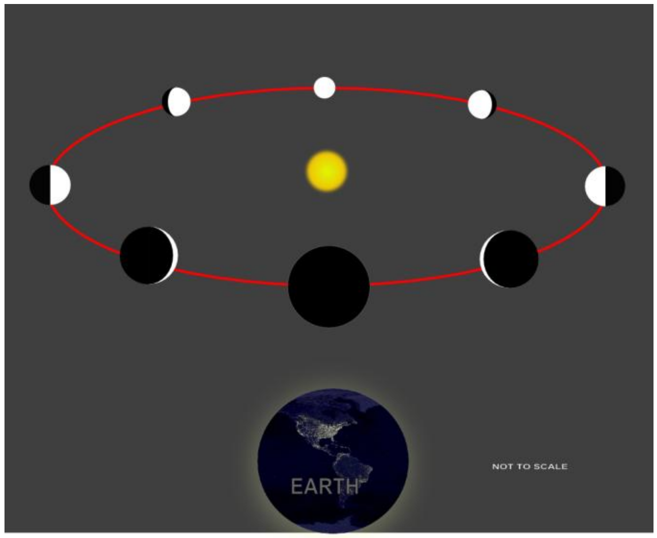


e. Similarly, Mercury and Venus sometimes undergo retrograde motion.

3. Copernicus was a conservative innovator. He still used perfect circles, and he did not seek a physical basis; his intellectual outlook was closer to that of Ptolemy than of Newton.
   a. To make the predicted and observed positions of planets agree, he needed to have epicycles superposed on the circles; moreover, the circles were not exactly centered on the Sun.
   b. The resulting quantitative predictions were not clearly superior to those of the Ptolemaic system.
   c. The appeal of the heliocentric theory was largely of a philosophical nature.
4. Copernicus was supported by the Catholic Church and dedicated his work to the Pope.
   a. He was aware of some of the radical implications of his model, developed around 1510.
   b. A disciple of his, Georg Rheicus, was responsible for getting his main book published (De Revolutionibus ---"*Concerning the Revolutions*" 1543). A copy was handed to deathbed.
   c. An unsigned preface by Andreas Osiander, a leading theologian and Lutheran preacher, incorrectly implies that Copernicus disavowed any true belief in the physical reality of his hypothesis.

B. In 1609-1610, Galileo Galilei was the first to use a telescope for systematic astronomical observations.
   1. His most important discovery of relevance to the heliocentric theory was threat Venus goes through a complete set of phases, just like the Moon.
   2. If Venus shines by reflecting light from the Sun, then the full set of phases is impossible in the Ptolemaic model.
      a. Venus was known to always be in the vicinity of the Sun, never opposite the Sun in in the night sky.
      b. In the Ptolemaic model, Venus is between Earth and the Sun, and it's epicycle does not cross the Sun's orbit.
      c. Therefore, Venus could only exhibit the "new" and crescent phases.

3. In the Copernican model, on the other hand, all phases are expected.
   a. Full and gibbous Venus would be seen when Venus is more distant than the Sun.
. b. Venus would then also appear smaller than during the crescent phase, as observed.



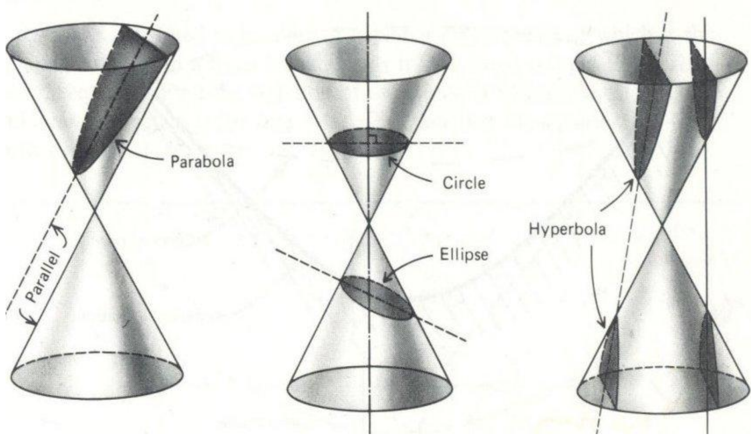
. 4. Galileo's observations of Venus where therefore a fatal blow to the Ptolemaic model.
5. Another of Galileo's major discoveries is that four bright moons orbit Jupiter.
   a. This showed that some heavenly bodies orbit other objects; Earth is not necessarily at the center.
. b. Moreover, since Jupiter was clearly moving, and it's moons were not left behind or flying off, Earth might be moving as well.
6. Galileo also made the following observations.
   a. The Moon has craters, mountains, and valleys, as well as "seas" (maria).
   b. The Sun has sunspots, which were considered "blemishes."
   c. Saturn has a complex shape (but he could not discern the rings).
   d. The Milky Way consists of a bunch of faint stars.
7. The Roman Catholic church was quite upset with Galileo's assertions that the
. Copernican model represents physical reality; he challenged the long-held belief
. in a "perfect", Earth-centered universe. Apparently, the Church also objected to
. his belief that matter consists of atoms.
   a. The Inquisition sentenced Galileo to perpetual house arrest for the remainder of his life.
   b. This had an important consequence; he devoted his time to the experimental study of the motions of falling bodies. Most of the principle in Newton's laws of motion are bases on experimental facts determined by Galileo.
   c. An important discovery made by Galileo is that objects of different mass accelerate at the same rate as they fall. Thus, a wooden ball and a much heavier lead ball dropped simultaneously also hit the ground simultaneously.
   d. One must neglect air resistance, of course; the experiment doesn't work with a hammer and a feather in air. However, it has been conducted dramatically on the

Moon (which lacks and atmosphere), as well as in vacuum chambers on Earth.

8. Pope John II informally pardoned Galileo in 1992, acknowledging that the Inquisition's Condemnation of Galileo had been too harsh.

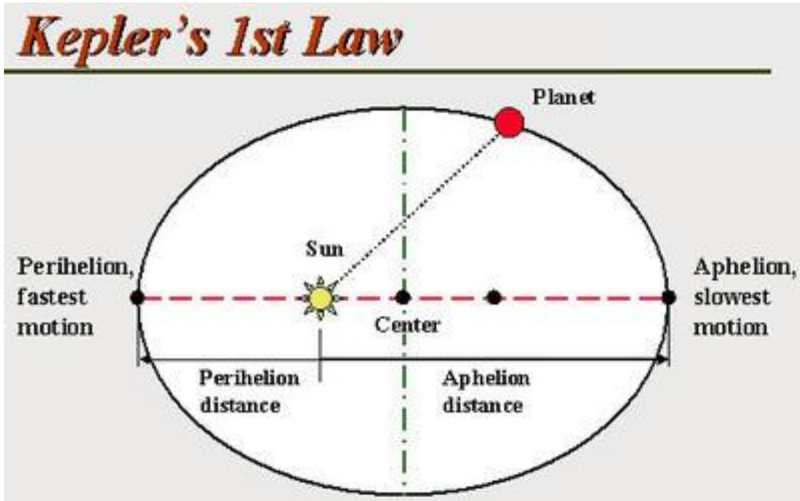
C. During the last 20 years of the 16th century, the Danish nobleman and astronomer Tycho Brahe used large instruments (but not telescopes) to measure the positions of Mars and other planets with unprecedented accuracy.

1. At the age of 14, he had decided to devote his life to astronomical observational after seeing a predicted eclipse of the Sun.
    a. He discovered a bright supernova in 1572, and showed that it is part of the "sphere of fixed stars", which is therefore not immutable.
    b. Most of his observations of planetary positions were conducted at his observatory Uraniborg on the island of Hveen.
2. In 1599, two year after losing his financial support in Denmark, he moved to Prague at the invitation of the Holy Roman Emperor, Rudolph II.
    a. A mathematically inclined assistant, Johannes Kepler, came to analyze the data. Kepler had previously tried and failed to explain the regularity of planetary orbits in terms of a nested sequence of the five "regular solids", which consist of equal sides (the pyramid, cube, octahedron, dodecahedron, and icosahedron).
    b. Tycho died in 1601, after Kepler had been with him for less than a year.
    c. Despite resistance from Tycho's relatives, Kepler gained access to Tycho's data, analysis of which led to three important revisions to the Copernican model.
    d. The first two of Kepler's laws were published in 1609, and the third in 1618.
    e. The laws are buried in Kepler's long, Baroque discourse on music and harmonics, but Isaac Newton read this and was able to state them succinctly.
3. Kepler's first law states that the orbits of planets are **ellipses**, not perfect circles, with the Sun at one focus of the ellipse (and nothing at the other focus).
    a. An ellipse is one type of conic section, produced by cutting the top off a hollow cone with a plane that is not perpendicular to the cone's axis.

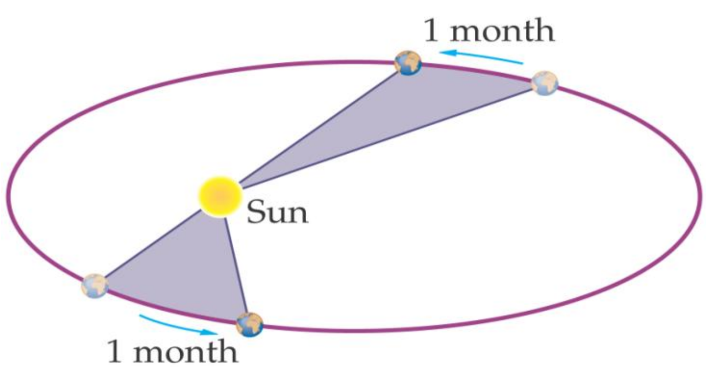


    b. Equivalently, an ellipse is a set of points having the following property; the sum of the distances from any two foci (fixed points), a + b, is a constant.

    c. The major and minor axes of an ellipse are the lengths of the longest and shortest axes, respectively. Half these lengths are the semimajor and semiminor axes. The semimajor axis is close to the average distance between a planet and the Sun.



   d. The eccentricity of an ellipse is the distance between the foci divided by it's major axis.
      i) Ellipses having the same major axis can have different eccentricities; just change the distance between the foci, keeping the sum a+b fixed.
      ii) If the two foci merge into one, the ellipse has zero eccentricity; it is a circle.
   e. One reason Tycho's accurate observations were crucial is that slightly off-center circles (as in the Copernican system) resemble the mildly eccentric orbits of most planets.
4. Kepler's second law states that a line joining the Sun and a planet sweep[s out equal areas in equal times.
   a. When a planet is close to the Sun, it moves faster than when it is far from the Sun; otherwise, the area swept out in a given time interval will be too small.
   b. An extreme example of this is the highly eccentric orbits of many comets; they spend very little time near the Sun.
5. Kepler's third law states that the square of a planet's orbital period is proportional to the cube of it's semimajor axis (or average distance from the Sun).
   a. In mathematical form, this is $P^2=kR^3$, where P is the orbital period, R is the semimajor axis, and k is a constant (the same for all planets).



(a)

   b. The more distant a planet from the Sun, the longer it takes to complete an orbit.
   c. Kepler was very pleased with this third law, which he called the "harmonic law". He had been incessantly searching for musical harmonics, and this law suggested an

almost musical relationship between orbital periods and sizes.

6. When discussing planets in our Solar System, it is sometimes convenient to use units based on Earth's orbit.
   a. We can write $P_p^2 = k\, R_p^2$, where the subscript "p" means that we are referring to a given planet.
   b. Similarly, we can write $P_e^2 = kR_e^3$, where the subscipts "e" means that we are referring to the Earth.
   c. Dividing one equation by the other, we see that the constant k cancels out: $(P_p/P_e)^2 = (R_p/R_e)^3$.
   d. If we now adopt units of years for P_p and A.U. for R_e, we have $P_p^2 = R_p^3$, since $P_e = 1$ year and $R_e = 1$ A.U.
   e. For example, if we know that the orbital period of Mars is 1.88 years (i.e., $P_p = 1.88$ in these units), then $1.88^2 = 3.53 = R_p^3$, so the semimajor axis of Mars's orbit is the cube root of 3.53, or 1.52 (i.e., $R_p = 1.52$ A.U.).
   f. Note that for nearly circular orbits, the semimajor axis is simply the orbital radius.
7. Kepler's three laws are purely empirical; Kepler had no physical explanation for their origin. Nevertheless, they were a major step in the development of astronomy.
8. Kepler's laws apply to any orbiting systems, not just the planets revolving about the Sun.
   a. For example, if we know that a satellite orbiting just above Earth's atmosphere (say, at an altitude of 250 km) has a period of about 90 minutes (1.5 hours), we can calculate the orbital radius R of a satellite whose period is 24 hours.
   b. The distance of the nearby satellite from the Earth's center is about 6630 km.
   c. Thus, we have $(24/1.5)^2 = (R/6630\text{ km})^3$
   d. Solving, we find R = 42,100 km, the radius of the distant satellite's orbit.
   e. Since Earth's radius is about 6380 km, the satellite is approx. 35,700 km above the Earth's surface.
   f. With a period of 24 hours, such satellites remain above a given point on Earth. They are called geostationary satellites, and have many uses (e.g. telecommunications).

D. Isaac Newton, born in 1642 (the year of Galileo's death), derived Kepler's laws mathematically from fundamental physical principles. To better appreciate this, we examine some of the contributions of this incredible scientist.

1. As mentioned earlier, Newton determined the "white light" consists of all colors of the rainbow, and he invented the reflecting telescope.
2. Among his even greater achievements were the development of the laws of motion, the law of universal gravitation, and the body of mathematics known as calculus.
3. Newton was reluctant to write up and publish his ideas. His good friend Edmund Halley (of Comet Halley fame) persuaded him to do so, and the result was a monumental book known as *The Principia* (1687).
4. Newton's three laws of motion, developed largely from an analysis of Galileo's data on falling bodies, are as follows.
   a. A body continues to be at rest, or in motion in a straight line with constant speed, unless a force acts on it.
      i) In particular, a planet doesn't need any force to keep it going.
   b. When a force acts on a body, it accelerates the body in the direction of the force, at a rate that is proportional to the strength of the force and inversely proportional to the mass of the body. (F = ma)
      i) Note that velocity refers to both speed and direction, and acceleration is the rate at which velocity changes—speed or direction (or both).
      ii) Pulling on a planet from the side changes the direction of motion. This is

how the gravitational force of the Sun keeps a planet curving around in it's orbit.

iii) A large mass is accelerated less than a small mass, for a given force.

c. When two bodies interact, they exert equal forces on each other, in opposite directions.

i) For every action, there is an equal and opposite reaction.

ii) A force always comes in pairs, and act in opposite directions.

iii) When you jump off a chair and the Earth's force of gravity brings you down again, you exert just as strong a force on the Earth as it does on you. It's the mass dependence in the second law that makes the difference; that force accelerates the enormously massive Earth far less than it accelerates you.

iv) The Earth and Moon exert equal forces on each other, but the Moon gets much more acceleration because it is only 1/80 as massive as Earth. (The Earth is accelerated, though, and it follows a little monthly orbit 1/80 as large as that of the Moon).

E. Newton's law of universal gravitation is also of fundamental importance.

1. According to legend, an apple fell on or near Newton, and he guessed that qualitatively the same force acts upon the Moon, making it fall toward the Earth. This is the Earth's gravitational force.

a. It was sensible to suppose that all bits of matter contribute to the gravitational force exerted by an object; hence, the Earth's gravitational force on the apple of mass m is probably proportional to Earth's total mass M.

b. Conversely, the gravitational force exerted on the Earth by the apple of mass m is probably proportional to M.

c. Since the forces are equal in strength, they must depend on the product of the two Masses: Mm.

d. This is also consistent with Galeleo's result that the acceleration of a falling body such as an apple is independent of it's mass m; Since F=ma, we see that m cancels out if the Earth's gravitational force on the apple is proportional to Mm. As expected, the acceleration does depend on the Earth's mass M.

e. If the Earth's gravitational force weakened with distance like light spreading out from a candle, then it's strength would decrease in proportion to the *inverse square* of distance, $1/d^2$. This was perhaps a guess, but it was reasonable, and it led to agreement with observations.

f. Overall, then, one can write the gravitational force exerted by the Earth of mass M on an apple of mass m as F is proportional to $-Mm/d^2$, (the negative sign is a reminder that the force is attractive rather than repulsive).

g. Generalizing to two objects of masses m_1 and m_2, and writing the proportionality as an equality, we have $F = -Gm_1m_2/d^2$, where G is now called Newton's constant of gravitation.

h. The value of G is difficult to measure accurately because gravity is a weak force. The best current result is $6.673 \times 10^{-8} cm^3/g\text{-}s^2$.

2. At the Earth's surface, the measured acceleration due to gravity is $9.80 \ m/s^2$. (This is Equal to $32 \ ft/s^2$.)

a. Newton used his newly developed rules of calculus to show that the gravitational force of the Earth acts as though all of the Earth's mass were concentrated in a point at it's center.

b. Thus, the relevant distance at Earth's surface is the radius, R, about 6400 km.

c. The Moon was known to be about 384,000 km from the Earth's center, a distance

60 times larger than R.

d. Hence, Newton calculated that the acceleration of the Moon should be $(1/60)^2$ equals 1/3600 times that at the Earth's surface, or 0.27 cm/s^2 = 0.0027 m/s^2.

e. This is indeed the acceleration the Moon must have to maintain it's orbit around the Earth. Newton's law of gravitation therefore seemed to work, at least for the Moon.

3. Newton was also aware of the moons orbiting Jupiter, and of stars that orbit each other, and of clusters of stars that appear to be bound together. Thus, it seemed that the law gravitation might apply **universally**.

F. One might wonder why we say that the Moon is falling toward the Earth. What makes it . orbit the Earth, instead of hitting?

1. Suppose there is no gravity. According to Newton's first law, launching the Moon tangentially (i.e. perpendicular to the direction toward the Earth) results in motion along a straight line at constant speed.
2. Now suppose gravity is present, but the Moon is released from rest. According to Newton's second law, the Moon accelerates toward Earth due to the force of gravity.
3. Let us combine these motions, and examine them along small time steps.

a. We see that the Moon falls toward the Earth a little during the time it takes to travel a short distance perpendicular to the Earth. It's new distance from the center of the Earth remains unchanged.

b. During the next step, the same thing happens, but now the tangential motion (perpendicular to the direction toward the Earth) is in a slightly different absolute direction, due to gravity, but it never reaches Earth because the tangential motion keeps it away.

c. If we imagine numerous tiny steps of time, we get a smooth, curved oribit. Newton showed this rigorously by using calculus.

d. Thus, the Moon rally does fall toward the Earth due to gravity, but it never reaches Earth because the tangential motion keeps it away.

e. The Moon presumably acquired it's tangential speed from the rotating disk of particles from which it formed.

4. If Earth's gravity were suddenly eliminated, the Moon would continue to move with Constant velocity (speed and direction) along the direction tangent to the orbit at at that instant.

G. Newton's reasoning is also illustrated by a drawing adapted from *The Principia*.

1. Imagine standing on a high tower or mountain, and ignore air resistance.
2. If you throw a rock straight out, initially parallel to the Earth's surface, it will fall to the ground some distance away from you due to gravity.
3. If you throw it faster, it will fall to the ground a larger distance away.
4. If you throw it very fast, it will eventually fall to the ground, but the distance will be even larger than naively expected because the surface of the Earth partially curves away from the rock's trajectory.
5. If you throw the rock sufficiently fast, the rock will follow a trajectory that exactly matches the curvature of the Earth, so it will not hit the ground; instead, it will orbit the Earth.
6. The requisite speed is about 8 km/s, somewhat lower than the escape speed form The Earth (11 km/s).
7. The orbital period close to the Earth's surface is about 90 minutes. This is the Period of the Hubble Space Telescope, for example.

H. Using his laws of motion and universal gravitation, Newton was able to derive and generalize Kepler's laws, a stunning achievement.

1. Kepler's laws apply to any objects attracted by gravity to any other objects; they are not limited to planetary orbits around the Sun.
    a. For example, they apply to the moons orbiting Jupiter.
    b. Kepler's third law, especially, will be encountered numerous times in this course.
2. The trajectories are conic sections, not limited only to ellipses.
    a. If an object is gravitationally bound to another object, the orbit is an ellipse (the curve formed when a plane intersects a hollow cone at an angle less steep than the side of the cone).
    b. If an object is unbound, the orbit is a hyperbola (the curve formed when a plane intersects a hollow cone at an angle steeper than the side of the cone).
    c. If an object is just barely unbound, it's orbit is a parabola (the curve formed when a plane intersects a hollow cone at an angle equal to the side of the cone).
3. The full version of Kepler's third law is actually $P^2=\{4\pi^2/[G(m_1 + m_2)]\}R^3$, where m_1 is the mass of the Sun, then m_2 is the mass of the planet.
    a. Note that the "constant" in Kepler's version of the third law therefore depends on m_2, the mass of the planet under consideration; it is not really a constant.
    b. However, since the mass of each planet in the Solar System is much smaller than the Sun's mass (Jupiter, with 0.001 times the Sun's mass, is by far the largest),the combination $4\pi^2/[G(m_1 + m_2)\}$ is nearly constant for all planets.
    c. Tycho's observations were not sufficiently accurate for Kepler to have noticed this slight dependence of the "constant" on the planet's mass.

I. If the mass m_2 is negligible relative to m_1, we can ignore m_2 in the sum (m_1+m_2).
   1. The generalized version of Kepler's third law then becomes $P^2 = [4\pi^2/(Gm_1)\}R^3$, where m_1 is the mass of the dominant object.
       a. If P and R of the small object (2) are measured, one can solve for m_1.
       b. For example, we can use this method to determine the Sun's mass.
           i) Assume the Earth's mass is negligible, and use the known values of P (1year) and R (1 A.U.) for the Earth.
           ii) Being careful to properly convert units, we find that the mass of the Sun is m_1=2 X 10^{33} grams.
           iii) This is indeed far larger than the mass of the Earth (m_2 = 6 X 10^{27} grams), thereby justifying the assumption that m_2 is negligible.
       c. Similarly, one can derive the mass of Jupiter relative to Earth by measuring the orbital speeds and distances of a few of it's bright moons.
   2. Another application of this is to determine the orbital speeds of different planets.
       a. For any planet, $P^2 = [4\pi^2/Gm_1)]R^3$.
       b. If the planet's orbit is circular (roughly true), then the circumference of the orbit ($2\pi R$) must equal the planet's speed multiplied by the period: $2\pi R = vP$. This is just an application of distance = speed x time, which is true for constant speed.
       c. Thus, $P = (2\pi R)/v$.
       d. Substituting this into equation: $P^2 = [4\pi^2/Gm_1)]R^3$, and rearranging, we find that $m_1 = v^2R/G$. We will see this again in our studies of galaxies.
       e. Equivalently, $v = (Gm_1/R)^{½}$, so we see that v is proportional to $(1/R)^{½}$.
       f. The speed of a planet is inversely proportional to the square root of it's semimajor axis ---i.e., distant planets move more slowly than those near the Sun.
       g. This relationship is characteristic of systems in which a large central mass dominates over the masses of orbiting particles.
       h. Later, we will compare this relationship with that found for rotating galaxies.

J. Questions:

1. Describe the heliocentric model proposed by Copernicus, and show how it explains the retrograde motion of planets.

3. Discuss the most important observational evidence Galileo found for the heliocentric hypothesis.
4. Summarize other key discoveries made by Galileo.

5. State Kepler's three laws of planetary motion.

6. Explain how Kepler's second law implies that a comet on a highly eccentric orbit spends very little time near the Sun.

7. Calculate the average distance of a planet from the Sun, given the planet's orbit period as 1.88 years.

8. Examining each of Galileo's telescopic discoveries separately, did they tend to support, oppose, or be irrelevant to the heliocentric hypothesis?
9. If planetary orbits can be reasonably approximated as circles centered on the Sun, can you use Kepler's third law to derive an equation relating the orbital speeds and distances of planets?

10. What do you think the response of the Pythagoreans would have been to Kepler's and Galileo's work?

11. State Newton's three laws of motion and give an example of each.

12. Describe Newton's law of universal gravitation, and summarize the reasoning used by Newton to deduce it.
13. Discuss the modifications Newton made to Kepler's three laws of planetary motion.

14. Explain why Kepler did not notice that his third law of planetary motion actually does depend on the mass of the planet under consideration.

15. Calculate the mass of the Sun from the orbital period of 1.88 years and distance of 1.5 A.U. of a planet.

16. Explain how we can say that the Moon is falling toward the Earth, when it clearly never reaches Earth.

17. Why does a satellite speed up as it spirals toward the Earth due to friction with the outer atmosphere? (Naively, it seems that the friction should cause it to slow down.)